Bayesian model comparison of solar flare spectra

J. Ireland¹ & G. D. Holman²

¹ADNET Systems, NASA's GSFC, Greenbelt MD, USA, ²NASA's GSFC, Greenbelt MD, USA

Introduction

We wish to determine which of two models is the preferred spectral description for a portion of RHESSI spectral data observed during the GOES X4.3 solar eruptive event of 23 July 2002. Solar flares accelerate electrons which can interact with the surrounding plasma to produce hard X-rays. For a general inhomogeneous optically thin source of plasma, bremsstrahlung photon flux energy density $F(E)$ in volume $V$ for electron energy $E$ can be written (Brown 1971) as

$$I(\varepsilon) = \frac{\pi^2}{4\varepsilon^2} \int_0^\infty T(L) Q(\varepsilon,E) dE,$$

where $Q(\varepsilon,E)$ is the bremsstrahlung cross-section differential in photon energy $\varepsilon$. We model the photon flux energy spectrum as the sum of emission due to a flare-irradiated electron spectrum interacting with a target, and emission from hot plasma with a Maxwellian distribution of speeds corresponding to some temperature $T$.

RHESSI observes incident hard X-ray flux $I$ by measuring the energy lost by the photon in the detector. This observed number of detector counts $D_i$ in energy bin $i \leq i \leq m$ is assumed to be drawn from a Poisson probability density function (PDF) $p(D_i | \mu)$ with mean $\mu_i$ in energy bin $i$ via a detector response matrix $R_i$ and incident photon flux $I$. By assuming a model spectrum for the accelerated electrons and a model for the interaction between it and the target plasma, the incident photon flux at RHESSI can be described by a parameter set $\theta$ (Eq. 3).

Hence for data $D = (D_0 \ldots D_m)$,

$$L(M|\theta, D) = \prod_{i=0}^{m} C_i^{D_i} e^{-C_i}$$

where $L$ is the likelihood of the data $D$ given a model $M$, parameterized by $\theta$, and information $I$.

We assume that the hard X-ray emission is well described by a thick target model, an isothermal background parameterized by emission measure [EM] and scaled temperature [KT] and a power law flare-irradiated electron spectrum that can include a single break, with low energy cutoff $E_0$, break energy $E_b$, and power-law indices $\delta_1$ and $\delta_2$ below and above $E_0$, respectively.

Two models for the observed emission

Two models are considered and fit to the data using OSPEX and Bayesian/MCMC model fitting.

Double power law (DPL) - 7 parameter fit

Thermal X-ray emission (Eq. 4) - EM, KT

Fermi-Dirac distribution in the product with a Gaussian distribution with mean and variance equal to the best fit from the previous iteration (starting with a 'best guess'). The fits obtained are shown in Figure 1.

The reduced-$\chi^2$ distribution has a deviation of $2(\Delta k)/\sigma = 0.11$ where $\Delta k = 1$ is the number of degrees of freedom in the fit for the model fits to the data. The minimum reduced-$\chi^2$ values found by fitting cannot distinguish between these two models within one deviation (Andrae et al. 2010).

Bayesian/MCMC model fitting

Bayes' Theorem states that

$$p(M|D, I) \propto p(D|M, I) p(M|I),$$

where $p(D|M, I)$ is the posterior PDF for the model $M$ given the data $D$. The quantity $p(I)$, $M$ is the prior probability that the model parameters have a given value, and $p(D|M, I)$ is a constant. We assume that each variable has a constant probability within a reasonable range (Tables 1, 2). A MCMC algorithm is used to sample the posterior PDF. Figure 2 shows that for both models all parameters are either moderately or strongly correlated / anti-correlated. Tables 1 & 2 show that some parameters have asymmetric uncertainty estimates.

Which model is preferred?

Bayes' Theorem can be used to calculate the odds ratio $O_{12}$ in favor of one model $1$ over model $2$. If model $1$ has prior probability $p(M_1|\beta)$ then

$$O_{12} = \frac{p(M_1|\beta)}{p(M_2|\beta)} = \frac{1}{1 + p(M_1|\beta) D_{12}},$$

where $D_{12}$ is the Bayes' factor,

$$D_{12} = L(M_1)/L(M_2),$$

is the global likelihood of the model $M_1$.

Assuming that we have no prior information to prefer either the NUI or DPL models we obtain

$$O_{\text{NUI/DPL}} \approx 13$$

Using the Kass & Raftery (1995) interpretation of odds ratio values, the NUI model is substantially to strongly preferred.

The Bayesian Information Criterion

$$BIC = -2 \ln \max(L) + k \ln N$$

is derived by suitably approximating the posterior close to its maximum. The lowest value of the BIC is the preferred model. Comparing the NUI and DPL models we find

$$BIC_{\text{NUI}} - BIC_{\text{DPL}} = -4.4$$

which is interpreted as positive evidence (Neath & Cavanaugh 2012) preferring NUI over DPL.

Conclusions

The odds ratio and the BIC suggest that the NUI model is the preferred model of the two considered here.

Future work applying Bayesian data analysis techniques to RHESSI data analysis will include other models of the X-ray emission, simultaneous fitting of the background spectrum with the model spectrum, and inclusion of data from multiple RHESSI detectors while taking into account systematic differences between these detectors.

Contact: Jack.Ireland@nasa.gov