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**“Bunched Black Swans” in Complex Geosystems**

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**ABSTRACT:** For all natural hazards, the question of when the next “big” event will hit is the classic “black swan” exposed by the fat-tails of extreme events. In the environmental sciences, such questions are often in terms of average “return period,” e.g., “Is 50 year flood the “Thomas water level a 5-6 years event?” Frequently, however, we also care about the emergence of catastrophic events, and whether the probability of several big events occurring in close succession is truly “independent.” In the black swan “Bunching,” a “big-event” of a “burst,” defined by Lipchitz and Carbone, 2009, if the “burst” occurs in the form of a fractal process, could in fact be a combined series of “independent” 5-6 years wide-bursting events.

Several available stochastic approaches provide quantitative information about such bursts, including: Extreme Value Theory (EVT), the theory of record level sets, spatial-temporal process and models of space-time “realizations” of activity in non-stationary systems. Some focus more on the possibility of single large events. Others are concerned with extended dwell times above a given approximately fractal threshold. The state of the art in this area has been deeply enriched by the growing literature on multifractal processes that are the natural extension of EVT. EVT is perhaps the best known of the paradigms. It is concerned with the distribution obeyed by the extremes of datasets, e.g., the 100 values obtained by considering the largest daily temperature in each of the years of a century. However, the pioneering work of Fama (1965), the “random walk” assumption for the series, long since found well known to most people that white Gaussian noise was the natural assumption. This has been deepened by models of heavy fat tails and long range dependence, allowing us to study how these effects combine in determining the burst duration and size exponent probability distributions.

In the presentation we compare EVT with multifractal process, the multifractal random walk (MRW). The presentation discusses two recent examples of our work on the burst problem, the above results seem to extend the previous results presented in (Watkins et al., 2006) with a standard self-similar model which fractional Brownian motion (fBm). fBm explicitly includes both heavy tails and long range dependence, allowing us to study how these effects combine in determining the burst duration and size exponent probability distributions. Here we use a multifractal process for the above results and make a qualitative comparison with that in the pdf probability distribution.

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**Linear Fractional Stable Motion, random walk-type model of how heavy tails & Joseph effect conspire to produce bursts**

Our motivation ...

A common thread across many complex systems is intermittent energy release events.

**“Black swans”, extremes, bursts ...**

But is knowing how often you see events best indicator of how long you’ll wait for next one?

- **No 1.** Well known to most people that white Gaussian or stable noises can show apparent clustering.
- **But more profoundly** magnitudes need not be independent … may be autocorrelated (e.g. AR1) or even long range dependent (“1/F”) ...

Black swans may be correlated ... gray swans may be bunched ...

Mandelbrot & Wallis (1968)

Long range dependence, the “Joseph effect”

“Physical swan” illustrates Pharaoh’s seven years of plenty (grain stores) in years of drought (brown stores). New shuffling sees "...activity burst concept, c.f. Bak et al’s Self Organised Criticality naturally interpolates between this & an individual spike.

Heavy tails & Joseph effect

- **Top trace** adds l.r.d to a-stable noise (H=0.75).
- **Lower trace** compares a Gaussian white noise

Alternative burst model?

- **Dependence sometimes seen not in amplitude but squared amplitude:** In the unsigned magnitudes. Natural counterpart of the “volatility bunching” seen in finance—multifractals capture this effect.

Scaling of multifractal bursts

- For fractional Brownian motion we had
  \[ \beta = 2 - H \]
  Replace time by a multifractal time scaling factor such that we get 
  \[ \beta = 1 - c (1 - H) \]
  Effect of multifractality here is to increase burst duration.

Simulations of light-tailed bursts

- Tail pdf of burst size L and duration T predicted to have exponent y=2/(E+H) & T=2/H respectively
  (Watkins et al, PRE, 2009).

Simulations of heavy-tailed bursts

- Watkins et al, PRE, 2009 found expressions also reasonable down to \( \alpha \approx 1.6 \), but to fail completely by \( \alpha = 1 \). Nonstationarity may be a conceptual issue, also technical ones, work in progress.

**Conclusions:**

- Assessing hazard from “gray swans” events needs consideration of not only relative frequency but also bunched.
- Motivated by data and by models like (SOC) what sought to unify these effects we have begun to look theoretically and numerically at bursts in rich toy multifractal (LFSM (Watkins et al, PRE, 2009)) and multifractal models.

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